Age to Age Factor Selection under Changing Development

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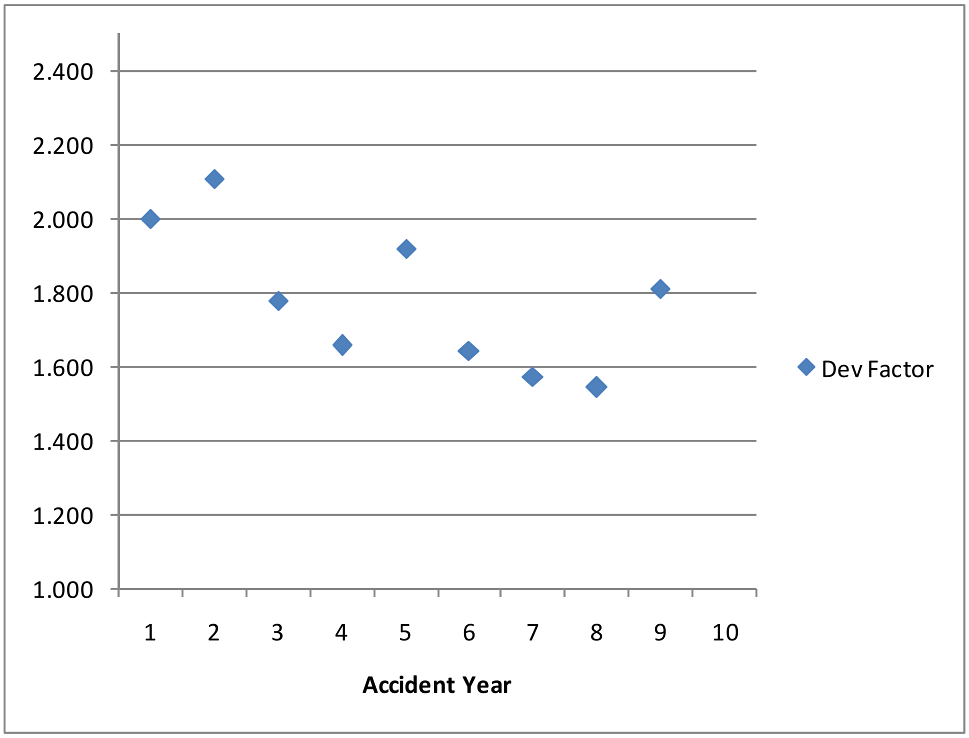
Introduction

A common question faced by many actuaries when selecting loss development factors is whether to base the selected factor on more recent data or to use a longer period of experience. If there are changes in the development patterns over time, a shorter period of experience may be more appropriate. If the experience is volatile, a longer period of experience may be more appropriate. Rarely is the case purely one or the other, but usually some combination of these influences. Most actuaries would agree on these basic principles, but the practical application of these principles often lacks analytical structure. This paper will suggest two systematic techniques intended to aid the actuary with this problem.

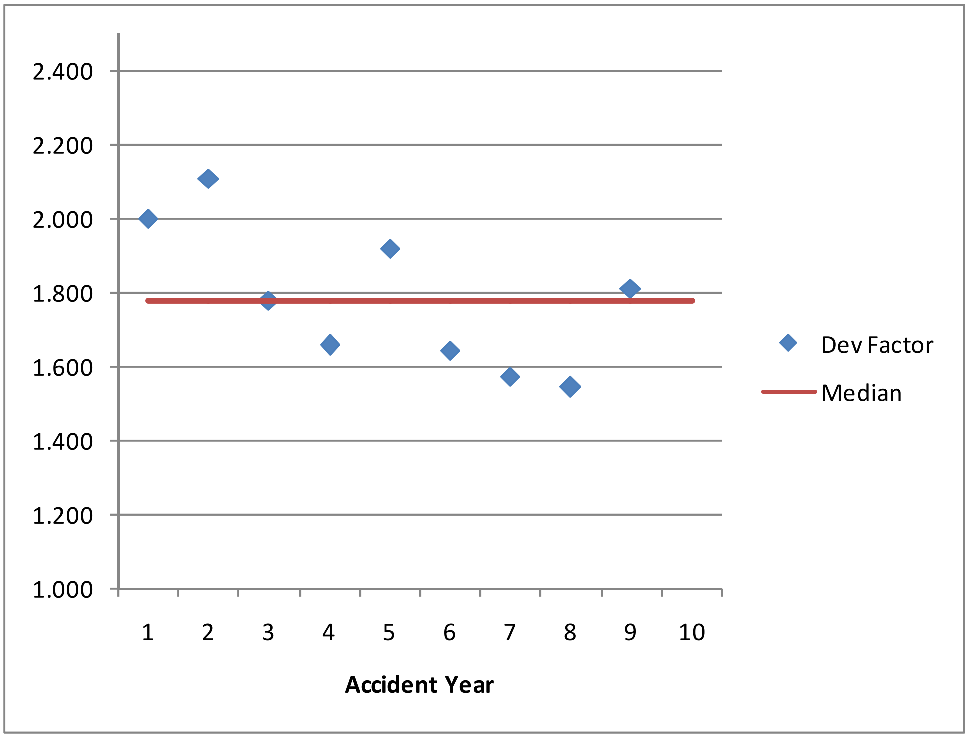
Change or Random Fluctuation?

At the center of this issue is the basic question of whether observed differences between current and past observed development factors are due to an underlying change or are due to random fluctuation (i.e. changes that are ephemeral in nature and not predictive of the future). Consider the following data for loss development factors from development age 1 to development age 2 for a number of accident years:

Year	Age 1 Loss	Age 2 Loss	Development Factor
1	29	58	2.000
2	28	59	2.107
3	36	64	1.778
4	35	58	1.657
5	25	48	1.920
6	31	51	1.645
7	35	55	1.571
8	35	54	1.543
9	21	38	1.810



Looking at the data points, there *appears* to be a trend and that we might be likely to expect a development factor for accident year 10 that is less than the long term average. But is this impression of changing development factors credible? One statistical test we can perform is a run test. If the data points are in fact randomly distributed from the same distribution, each point will be as likely to be above the median value as below it (regardless of the distribution). In data with a trend, there will be a tendency for the values that are above the median to be more likely at either end of the data series and fewer runs would result. Adding the median value to the data for this example we have the following:



We can label the series as (++-+---+). The third point, which is exactly at the median, is discarded. This constitutes five "runs" above and below the median. The probability distribution under randomness (i.e. zero trend) given nine data points is as follows:

Runs	Probability	Cumulative Probability
2	2.86%	2.86%
3	8.57%	11.43%
4	25.71%	37.14%
5	25.71%	62.86%
6	25.71%	88.57%
7	8.57%	97.14%
8	2.86%	100.00%
9	0.00%	100.00%

So according to this test, the observed pattern of data points is highly likely under the scenario where there is actually no underlying trend, despite the appearance to the contrary. A larger table with varying number of data points is included at the end of this paper.

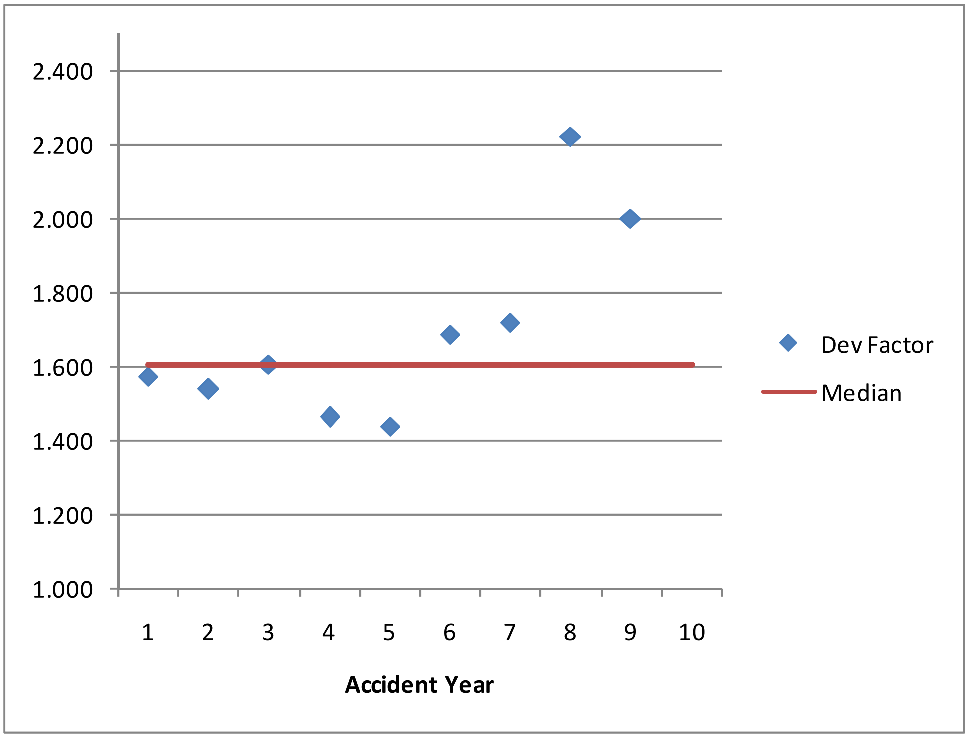
Another test we could run is to perform a regression and test the slope parameter for significance. Using simple linear regression we arrive at the following parameters:

	estimate	standard error	significance level
slope	-0.048	0.020	0.049
intercept	2.021	0.113	0.000

This analysis suggests that there is a non-zero trend. Why do these two tests provide inconsistent results? The run test, since it is based purely on whether observations are above or below the median is very robust (i.e. resistant to outliers) and is free of assumptions about the form of the probability distribution. The t-test of the regression parameter on the other hand assumes that the residuals from the linear relationship are independent random variables drawn from the same normal distribution N(0,σ). If the assumptions made by the regression test are correct then the observations are inconsistent with zero trend in the factors. The amount by which the individual data points deviate from the mean has an important impact. The run test does not consider the size of these deviations. The *amount* by which they are less or more than the median does not matter only that they are less or more than the median, because the run test has no distributional framework within which to place them.

Consider another example:

Year	Age 1 Loss	Age 2 Loss	Development Factor
1	42	66	1.571
2	39	60	1.538
3	38	61	1.605
4	39	57	1.462
5	41	59	1.439
6	35	59	1.686
7	32	55	1.719
8	27	60	2.222
9	23	46	2.000



In this example, the number of runs is 2 (----,++++), which is unlikely under the randomness assumption. The parameters of a linear regression are:

	estimate	standard error	significance level
slope	0.070	0.024	0.022
intercept	1.342	0.135	0.000

In this example the two tests give consistent indications regarding changes in the development factor.

The "Observed" Data

To this point, no mention has been made about the creation of the data points for these two examples. **Both of these examples were created from the same underlying simulated random process with the same parameters.** Specifically the following random process was simulated:

- 100 claim payments all of identical size
- Each claim is paid at a random "age" of development according to an exponential distribution (i.e. the lag from the beginning of the accident year to the claim payment is exponentially distributed)
- The parameter for the exponential distribution varies from accident year to accident year

Accident Year	Expected Lag	Implied 1-2 Development Factor
1	2.000	1.607
2	2.100	1.621
3	2.205	1.635
4	2.315	1.649
5	2.431	1.663
6	2.553	1.676
7	2.680	1.689
8	2.814	1.701
9	2.955	1.713
10	3.103	1.724

The implied (true) development factor is calculated as the ratio CDF(2)/CDF(1) = $(1-e^{-2/lag})/(1-e^{-1/lag})$.

The simulated claim payments were summarized by accident year and development period, and the "observed" age to age factors calculated.

Practical Application of Significance Tests

As a practical reality we as actuaries do not have the luxury of saying, "The data is inconclusive." and moving on to the next academic study. We are required to use our best judgment to predict the future. This judgment includes looking at data, but also reflects knowledge of changes in claims practices, underlying tort environment, etc. Unfortunately, however, this judgment can be biased by the data itself. An *apparent* change causes us to *look* for reasons for the change, justifying an impression that may be misleading. Statistical testing can help us to recognize whether or not a change is truly significant, but we are left with many choices regarding the nature of tests and the assumptions made by these tests. This paper will now present two methods to assist with this problem, each with flexibility to allow the actuary to adjust judgmentally the amount of stability or responsiveness to observed fluctuations, but still within general framework of statistics.

Method 1- Run Test Data Reduction

This method is simply built on the run test described above, eliminating data until the randomness hypothesis is not rejected.

Step 1—Determine a significance parameter between 0 and 1. (0=least responsive, most stable, 1 = most responsive, least stable).

Step2—Calculate the median of all accident years' factors

Step3—Perform a run test. Reject the randomness hypothesis if the cumulative probability of observing this number of runs is below the significance level selected in step 1.

Step4—If the randomness hypothesis was rejected in Step 3, eliminate the oldest observation and repeat Step 2-Step4 until the hypothesis is not rejected.

Step5—If the randomness hypothesis was rejected in Step 3, calculate the average (volume weighted or simple) of the remaining observations.

Method 2- Stabilized Regression

This method uses regression, but expands upon the simple linear regression described above. It is volume weighted and uses logarithms to adjust for the typical skewness of observed age to age factors. It reflects trend in development factors only to the extent the trend is considered statistically significant. The amount of trend that is used tempered based on this significance test. It also defaults to the long term weighted average factor when the trend is not considered statistically significant. It minimizes weighted squared residuals, subject to a constraint of balance (i.e. the total projected loss across all historical periods is equal to the total actual loss).

Let X_{ij} denote the cumulative loss for accident year i at development period j.

Step 1—Determine a significance parameter θ between 0 and 1. (0=least responsive, most stable, 1 = most responsive, least stable).

Step 2—Calculate the weighted average development factor

$$W = \frac{\sum_i X_{i,j+1}}{\sum_i X_{i,j}}$$

Step 3—Calculate the Average Accident Year

$$A = \frac{\sum_i iX_{i,j}}{\sum_i X_{i,j}}$$

Step 4—Calculate Raw Slope

$$S1 = \frac{\sum_i X_{i,j}(i-A)\left(ln\left({X_{i,j+1}}/{(WX_{i,j})}\right)\right)}{\sum_i X_{i,j}(i-A)^2}$$

Step 5—Calculate Raw Intercept

$$I1 = ln(W) - S1 \cdot A$$

Step 6—Calculate Standard Error of the Raw Slope

$$Se = \frac{\sqrt{\sum_i X_{i,j}^2 (i-A)^2}\sqrt{\frac{\sum_i X_{i,j}\left(I1 + S1 \cdot i - ln\left({X_{i,j+1}}/{(X_{i,j})}\right)\right)^2}{\sum_i X_{i,j}}}}{\sum_i X_{i,j}(i-A)^2}$$

Step 7—Calculate Stability Threshold for Slope

$$T = Se \cdot CDF_{t\,dist}(1-\theta/2)$$

Step 8—If |S1| > T and S1 > 0 then set tempered slope S = S1 – T.
If |S1| > T and S1 < 0 then set tempered slope S = S1 + T.
If |S1| < T then set tempered slope S = 0.

Step 9—Calculate Intercept consistent with slope S.

$$I = ln(W) - S \cdot A$$

Step 10—Calculate the projected development factor for Accident Year i as:

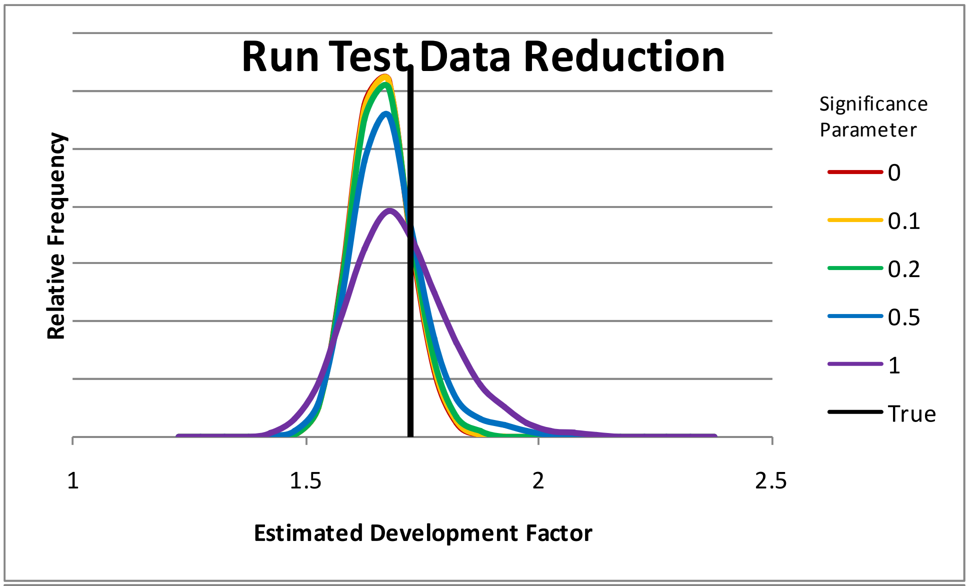
$$e^{I+iS}$$

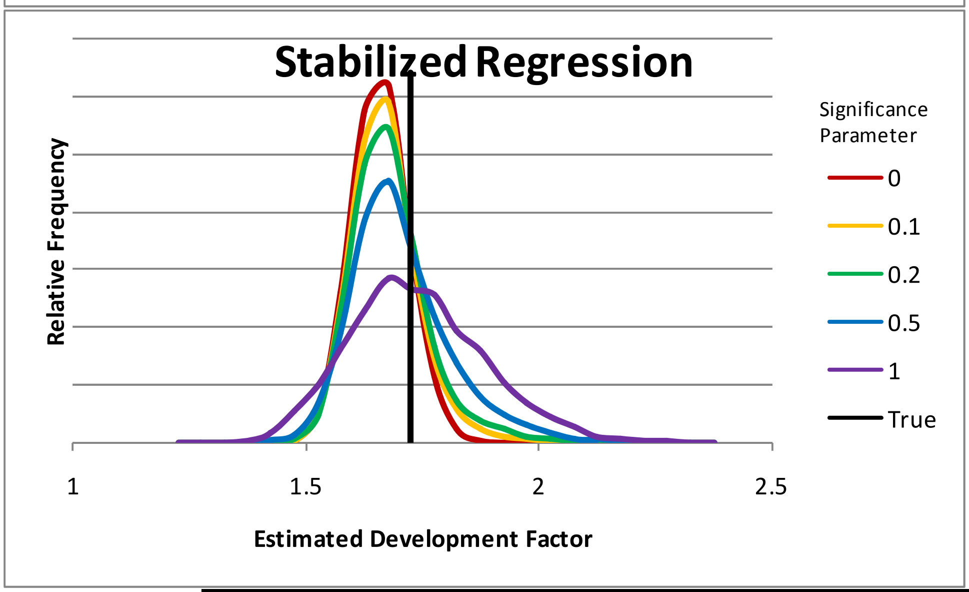
Applying each of these methods to the two examples shown above to estimate the development factor to use for Accident Year 10 gives the following table:

Projected Development Factor for Accident Year 10

	Example 1		Example 2	
Significance Parameter	Method 1	Method 2	Method 1	Method 2
0.000	1.764	1.764	1.655	1.655
0.100	1.764	1.667	1.689	1.769
0.200	1.764	1.629	1.766	1.825
0.300	1.764	1.606	1.766	1.860
0.400	1.764	1.589	1.963	1.887
0.500	1.764	1.574	1.963	1.910
0.600	1.764	1.562	1.963	1.930
0.700	1.615	1.551	1.963	1.948
0.800	1.615	1.541	1.963	1.966
0.900	1.615	1.531	1.963	1.983
1.000	1.615	1.521	1.963	1.999

The true factor is 1.724. Both of the methods return the loss weighted average factor (all data points) when the significance parameter is set to 0. These are only two examples. Simulating 10,000 times we have the following results:





	Run Test Data Reduction			Stabilized Regression		
Significance Parameter	Average	Bias	RMSE	Average	Bias	RMSE
0	1.660	-0.064	0.088	1.660	-0.064	0.088
0.1	1.661	-0.063	0.088	1.672	-0.053	0.091
0.2	1.664	-0.061	0.088	1.680	-0.044	0.095
0.5	1.677	-0.047	0.094	1.703	-0.021	0.110
1	1.703	-0.022	0.111	1.739	0.014	0.148

The full triangle weighted average (significance parameter =0 for both methods) is of course negatively biased for this example of increasing development lags. Not surprisingly the run test data reduction method never is able to eliminate its negative bias in this scenario, since it will never extrapolate. The stabilized regression method, since it does extrapolate, eventually eliminates the negative bias, but at a cost of a significantly increased root mean squared error.

These results are specific to the simulated claims process that was used. The results of each of these methods would obviously vary depending on the actual underlying claim process, which of course is unknown, including non-monotonic shifts, etc. The actual choice of a significance parameter to use in either method could be based on the perceived potential for changes in claims handling, mix of business, legal environment, etc.

Cumulative Probability of Number of Runs for a Particular Number of Observations

Number of Observations	Number of Runs: 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	1.0000																		
3	1.0000																		
4	0.3333	0.6667	1.0000																
5	0.3333	0.6667	1.0000																
6	0.1000	0.3000	0.7000	0.9000	1.0000														
7	0.1000	0.3000	0.7000	0.9000	1.0000														
8	0.0286	0.1143	0.3714	0.6286	0.8857	0.9714	1.0000												
9	0.0286	0.1143	0.3714	0.6286	0.8857	0.9714	1.0000												
10	0.0079	0.0397	0.1667	0.3571	0.6429	0.8333	0.9603	0.9921	1.0000										
11	0.0079	0.0397	0.1667	0.3571	0.6429	0.8333	0.9603	0.9921	1.0000										
12	0.0022	0.0130	0.0671	0.1753	0.3918	0.6082	0.8247	0.9329	0.9870	0.9978	1.0000								
13	0.0022	0.0130	0.0671	0.1753	0.3918	0.6082	0.8247	0.9329	0.9870	0.9978	1.0000								
14	0.0006	0.0041	0.0251	0.0775	0.2086	0.3834	0.6166	0.7914	0.9225	0.9749	0.9959	0.9994	1.0000						
15	0.0006	0.0041	0.0251	0.0775	0.2086	0.3834	0.6166	0.7914	0.9225	0.9749	0.9959	0.9994	1.0000						
16	0.0002	0.0012	0.0089	0.0317	0.1002	0.2145	0.4048	0.5952	0.7855	0.8998	0.9683	0.9911	0.9988	0.9998	1.0000				
17	0.0002	0.0012	0.0089	0.0317	0.1002	0.2145	0.4048	0.5952	0.7855	0.8998	0.9683	0.9911	0.9988	0.9998	1.0000				
18	0.0000	0.0004	0.0030	0.0122	0.0445	0.1090	0.2380	0.3992	0.6008	0.7620	0.8910	0.9555	0.9878	0.9970	0.9996	1.0000	1.0000		
19	0.0000	0.0004	0.0030	0.0122	0.0445	0.1090	0.2380	0.3992	0.6008	0.7620	0.8910	0.9555	0.9878	0.9970	0.9996	1.0000	1.0000		
20	0.0000	0.0001	0.0010	0.0045	0.0185	0.0513	0.1276	0.2422	0.4141	0.5859	0.7578	0.8724	0.9487	0.9815	0.9955	0.9990	0.9999	1.0000	1.0000
21	0.0000	0.0001	0.0010	0.0045	0.0185	0.0513	0.1276	0.2422	0.4141	0.5859	0.7578	0.8724	0.9487	0.9815	0.9955	0.9990	0.9999	1.0000	1.0000
22	0.0000	0.0000	0.0003	0.0016	0.0073	0.0226	0.0635	0.1349	0.2599	0.4100	0.5900	0.7401	0.8651	0.9365	0.9774	0.9927	0.9984	0.9997	1.0000
23	0.0000	0.0000	0.0003	0.0016	0.0073	0.0226	0.0635	0.1349	0.2599	0.4100	0.5900	0.7401	0.8651	0.9365	0.9774	0.9927	0.9984	0.9997	1.0000
24	0.0000	0.0000	0.0001	0.0005	0.0028	0.0095	0.0296	0.0699	0.1504	0.2632	0.4211	0.5789	0.7368	0.8496	0.9301	0.9704	0.9905	0.9972	0.9995
25	0.0000	0.0000	0.0001	0.0005	0.0028	0.0095	0.0296	0.0699	0.1504	0.2632	0.4211	0.5789	0.7368	0.8496	0.9301	0.9704	0.9905	0.9972	0.9995
26	0.0000	0.0000	0.0000	0.0002	0.0010	0.0038	0.0131	0.0341	0.0812	0.1566	0.2772	0.4179	0.5821	0.7228	0.8434	0.9188	0.9659	0.9869	0.9962
27	0.0000	0.0000	0.0000	0.0002	0.0010	0.0038	0.0131	0.0341	0.0812	0.1566	0.2772	0.4179	0.5821	0.7228	0.8434	0.9188	0.9659	0.9869	0.9962
28	0.0000	0.0000	0.0000	0.0001	0.0004	0.0015	0.0056	0.0157	0.0412	0.0871	0.1697	0.2798	0.4266	0.5734	0.7202	0.8303	0.9129	0.9588	0.9843
29	0.0000	0.0000	0.0000	0.0001	0.0004	0.0015	0.0056	0.0157	0.0412	0.0871	0.1697	0.2798	0.4266	0.5734	0.7202	0.8303	0.9129	0.9588	0.9843
30	0.0000	0.0000	0.0000	0.0000	0.0001	0.0006	0.0023	0.0070	0.0199	0.0457	0.0974	0.1749	0.2912	0.4241	0.5759	0.7088	0.8251	0.9026	0.9543
31	0.0000	0.0000	0.0000	0.0000	0.0001	0.0006	0.0023	0.0070	0.0199	0.0457	0.0974	0.1749	0.2912	0.4241	0.5759	0.7088	0.8251	0.9026	0.9543
32	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002	0.0009	0.0030	0.0092	0.0228	0.0528	0.1028	0.1862	0.2933	0.4311	0.5689	0.7067	0.8138	0.8972
33	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002	0.0009	0.0030	0.0092	0.0228	0.0528	0.1028	0.1862	0.2933	0.4311	0.5689	0.7067	0.8138	0.8972
34	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0003	0.0012	0.0041	0.0109	0.0272	0.0572	0.1122	0.1907	0.3028	0.4290	0.5710	0.6972	0.8093
35	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0003	0.0012	0.0041	0.0109	0.0272	0.0572	0.1122	0.1907	0.3028	0.4290	0.5710	0.6972	0.8093
36	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0005	0.0017	0.0050	0.0134	0.0303	0.0640	0.1171	0.2004	0.3046	0.4349	0.5651	0.6954
37	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0005	0.0017	0.0050	0.0134	0.0303	0.0640	0.1171	0.2004	0.3046	0.4349	0.5651	0.6954
38	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002	0.0007	0.0022	0.0064	0.0154	0.0349	0.0683	0.1256	0.2044	0.3127	0.4331	0.5669
39	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002	0.0007	0.0022	0.0064	0.0154	0.0349	0.0683	0.1256	0.2044	0.3127	0.4331	0.5669
40	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0003	0.0009	0.0029	0.0075	0.0182	0.0380	0.0748	0.1301	0.2130	0.3143	0.4381